FINDINGS:

The code performs PCA on the USArrests dataset using the prcomp() function with scale = TRUE to standardize the variables. The summary() function displays information on the variance explained by each principal component, as well as the loadings of the variables on each component.

The standard deviations of the four principal components are displayed in the first row of the summary, with the first principal component explaining the most variance (1.57 units) and the fourth component explaining the least (0.56 units). Together, the four components explain all the variance in the data.

The rotation matrix displayed in the second row of the summary gives the loadings of each variable on each component. For example, the loading of Murder on PC1 is -0.5359, indicating that states with high murder rates tend to have low scores on PC1. The loading of UrbanPop on PC2 is -0.8728, meaning that states with high percentages of urban population tend to have low scores on PC2.

The interpretation of each principal component depends on the loadings of the variables. In this case, PC1 can be interpreted as a measure of overall crime rate, as it is positively correlated with all variables in the dataset. PC2 can be interpreted as a measure of urbanization, as it is negatively correlated with UrbanPop. PC3 is a measure of sexual assault, as it is strongly positively correlated with Rape. PC4 is a measure of the difference between violent and non-violent crimes, as it is positively correlated with Murder and negatively correlated with Assault

CODE:

data(USArrests) states <- row.names(USArrests) names(USArrests) apply(USArrests, 2, mean) apply(USArrests, 2, var) pr.out <- prcomp(USArrests, scale = TRUE) names(pr.out)

pr.out\$center

pr.out\$scale

pr.out\$rotation

dim(pr.out\$x)

biplot(pr.out, scale = 0)

pr.out\$rotation = -pr.out\$rotation

pr.out = -pr.out x

```
biplot(pr.out, scale = 0)
```

pr.out\$sdev

pr.var <- pr.out\$sdev^2

pve <- pr.var / sum(pr.var)</pre>

par(mfrow = c(1, 2))

plot(pve, xlab = "Principal Component",

ylab = "Proportion of Variance Explained", ylim = c(0, 1),

type = "b")

plot(cumsum(pve), xlab = "Principal Component",

ylab = "Cumulative Proportion of Variance Explained",

ylim = c(0, 1), type = "b")

a <- c(1, 2, 8, -3)

cumsum(a)

X <- data.matrix(scale(USArrests))

pcob <- prcomp(X)</pre>

summary(pcob)

sX <- svd(X)

names(sX)

round(sX\$v, 3)

pcob\$rotation

```
t(sX$d * t(sX$u))
pcob$x
nomit <- 20
set.seed(15)
ina <- sample(seq(50), nomit)
inb <- sample(1:4, nomit, replace = TRUE)
Xna <- X
index.na <- cbind(ina, inb)
Xna[index.na] <- NA
fit.svd <- function(X, M = 1) {
 svdob <- svd(X)</pre>
 with(svdob,
    u[, 1:M, drop = FALSE] %*%
     (d[1:M] * t(v[, 1:M, drop = FALSE]))
)
}
Xhat <- Xna
```

```
xbar <- colMeans(Xna, na.rm = TRUE)

Xhat[index.na] <- xbar[inb]

thresh <- 1e-7

rel_err <- 1

iter <- 0

ismiss <- is.na(Xna)

mssold <- mean((scale(Xna, xbar, FALSE)[!ismiss])^2)

mss0 <- mean(Xna[!ismiss]^2)
```

```
while(rel_err > thresh) {
```

```
iter <- iter + 1
Xapp <- fit.svd(Xhat, M = 1)
Xhat[ismiss] <- Xapp[ismiss]
mss <- mean(((Xna - Xapp)[!ismiss])^2)
rel_err <- (mssold - mss) / mss0
mssold <- mss
cat("Iter:", iter, "MSS:", mss,
    "Rel. Err:", rel_err, "\n")
}</pre>
```

cor(Xapp[ismiss], X[ismiss])

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|-------------------------------------------------------------------------------------|---------------------------|------------------------|---------------------|-----|
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| <pre>> data(USArrests)</pre> | | | | |
| <pre>> states <- row.names(USArrests)</pre> | | | | |
| > states | | | | |
| [1] "Alabama" "Alaska" | "Arizona" | "Arkansas" | "California" | |
| [6] "Colorado" "Connecticut" [11] "Hawaii" "Idaho" | "Delaware" | "Florida" "Indiana" | "Georgia" "Iowa" | |
| [11] "Hawaii" "Idaho" [16] "Kansas" "Kentucky" | "Illinois" "Louisiana" | "Indiana" "Maine" | "Maryland" | |
| [21] "Massachusetts" "Michigan" | "Minnesota" | "Mississippi" | "Missouri" | |
| [26] "Montana" "Nebraska" | "Nevada" | "New Hampshire" | "New Jersey" | |
| [31] "New Mexico" "New York" | "North Carolina" | | "Ohio" | |
| [36] "Oklahoma" "Oregon" | "Pennsylvania" | "Rhode Island" | "South Carolina" | |
| [41] "South Dakota" "Tennessee" | "Texas" | "Utah" | "Vermont" | |
| [46] "Virginia" "Washington" | "West Virginia" | "Wisconsin" | "Wyoming" | |
| <pre>> names(USArrests)</pre> | | | | |
| [1] "Murder" "Assault" "UrbanPop" | "Rape" | | | |
| <pre>> apply(USArrests, 2, mean)</pre> | | | | |
| Murder Assault UrbanPop Rape | | | | |
| 7.788 170.760 65.540 21.232 > apply(USArrests, 2, var) | | | | |
| Murder Assault UrbanPop | Rape | | | |
| | .72916 | | | |
| <pre>> pr.out <- prcomp(USArrests, scale =</pre> | | | | |
| > names(pr.out) | | | | |
| [1] "sdev" "rotation" "center" | "scale" "x" | | | |
| <pre>> pr.out\$center</pre> | | | | |
| Murder Assault UrbanPop Rape | | | | |
| 7.788 170.760 65.540 21.232 | | | | |
| <pre>> pr.out\$scale Murder Assault UrbanPop Ra</pre> | | | | |
| Murder Assault UrbanPop Ra 4.355510 83.337661 14.474763 9.3663 | pe 85 | | | |
| <pre>> pr.out\$rotation</pre> | 65 | | | |
| PC1 PC2 | PC3 PC4 | | | |
| Murder -0.5358995 0.4181809 -0.341 | 2327 0.64922780 | | | |
| Assault -0.5831836 0.1879856 -0.268 | 1484 -0.74340748 | | | |
| UrbanPop -0.2781909 -0.8728062 -0.378 | | | | |
| Rape -0.5434321 -0.1673186 0.817 | 7779 0.08902432 | | | |
| > dim(pr.out\$x) | | | | |
| $\begin{bmatrix} 1 \end{bmatrix} 50 4$ | | | | |
| <pre>> biplot(pr.out, scale = 0) > pr.out\$rotation = -pr.out\$rotation</pre> | | | | |
| > pr.out $rotation = -pr.out$ | | | | - |
| S birowchy - birowchy | | | | |

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| 🔞 R4.2.2 · ~/ 🔅 | |
| <pre>> pr.out\$sdev [1] 1.5748783 0.9948694 0.5971291 0.4164494 > pr.var <- pr.out\$sdev^2 > pr.var</pre> | • |
| <pre>[1] 2.4802416 0.9897652 0.3565632 0.1734301 > pve <- pr.var / sum(pr.var) > pve [1] 0.62006020 0.24744120 0.08014080 0.04225752</pre> | |
| <pre>[1] 0.62006039 0.24744129 0.08914080 0.04335752 > par(mfrow = c(1, 2)) > plot(pve, xlab = "Principal Component", + ylab = "Proportion of Variance Explained", ylim = c(0, 1), + type = "b")</pre> | |
| <pre>> plot(cumsum(pve), xlab = "Principal Component", + ylab = "Cumulative Proportion of Variance Explained", + ylim = c(0, 1), type = "b") > a <- c(1, 2, 8, -3)</pre> | ł |
| <pre>> cumsum(a) [1] 1 3 11 8 > X <- data.matrix(scale(USArrests)) > pcob <- prcomp(X)</pre> | |
| <pre>> summary(pcob) Importance of components:</pre> | |
| Standard deviation 1.5749 0.9949 0.59713 0.41645 Proportion of Variance 0.6201 0.2474 0.08914 0.04336 Cumulative Proportion 0.6201 0.8675 0.95664 1.00000 > sX <- svd(X) | |
| <pre>> names(sX) [1] "d" "u" "v" > round(sX\$v, 3) [,1] [,2] [,3] [,4]</pre> | |
| [1,] -0.536 0.418 -0.341 0.649 [2,] -0.583 0.188 -0.268 -0.743 [3,] -0.278 -0.873 -0.378 0.134 [4,] -0.543 -0.167 0.818 0.089 > pcob\$rotation | |
| PC1 PC2 PC3 PC4 Murder -0.5358995 0.4181809 -0.3412327 0.64922780 Assault -0.5831836 0.1879856 -0.2681484 -0.74340748 UrbanPop -0.2781909 -0.8728062 -0.3780158 0.13387773 Rape -0 5434321 -0 1673186 0.8177779 0.08902432 | Ŧ |

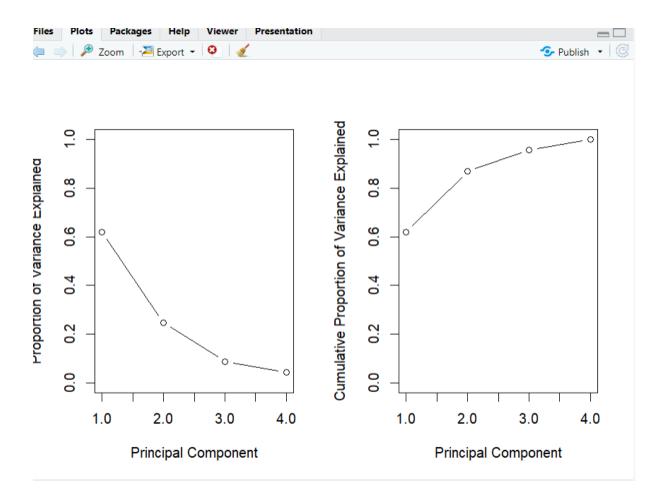
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|-----------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
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| > t(sX\$d * t(sX\$u)) | |
| [,1] [,2] [,3] [,4] | |
| [1,] -0.97566045 1.12200121 -0.43980366 0.154696581 | |
| [2,] -1.93053788 1.06242692 2.01950027 -0.434175454 | |
| [3,] -1.74544285 -0.73845954 0.05423025 -0.826264240 | |
| [4,] 0.13999894 1.10854226 0.11342217 -0.180973554 | |
| [5,] -2.49861285 -1.52742672 0.59254100 -0.338559240 | |
| [6,] -1.49934074 -0.97762966 1.08400162 0.001450164 | |
| [7,] 1.34499236 -1.07798362 -0.63679250 -0.117278736 | |
| [8,] -0.04722981 -0.32208890 -0.71141032 -0.873113315 | |
| [9,] -2.98275967 0.03883425 -0.57103206 -0.095317042 | |
| [10,] -1.62280742 1.26608838 -0.33901818 1.065974459 | |
| [11,] 0.90348448 -1.55467609 0.05027151 0.893733198 | |
| [12,] 1.62331903 0.20885253 0.25719021 -0.494087852 [12,] 1.2555107 0.57408544 0.57058547 0.100704015 | |
| [13,] -1.36505197 -0.67498834 -0.67068647 -0.120794916 [14,] 0.50039122 0.15003026 0.22576277 0.400207505 | |
| [14,] 0.50038122 -0.15003926 0.22576277 0.420397595 [15,] 2.23099579 -0.10300828 0.16291036 0.017379470 | |
| [15,] 2.23099579 -0.10300828 0.16291036 0.017379470 [16,] 0.78887206 -0.26744941 0.02529648 0.204421034 | |
| [17,] 0.74331256 0.94880748 -0.02808429 0.663817237 | |
| [17,] 0.74551250 0.84800748 -0.02808429 0.005817257 [18,] -1.54909076 0.86230011 -0.77560598 0.450157791 | |
| [19,] 2.37274014 0.37260865 -0.06502225 -0.327138529 | |
| [20,] -1.74564663 0.42335704 -0.15566968 -0.553450589 | |
| 21,] 0.48128007 -1.45967706 -0.60337172 -0.177793902 | |
| [22,] -2.08725025 -0.15383500 0.38100046 0.101343128 | |
| 23, 1.67566951 -0.62590670 0.15153200 0.066640316 | |
| 24, -0.98647919 2.36973712 -0.73336290 0.213342049 | |
| 25, -0.68978426 -0.26070794 0.37365033 0.223554811 | |
| 26, 1.17353751 0.53147851 0.24440796 0.122498555 | |
| [27,] 1.25291625 -0.19200440 0.17380930 0.015733156 | |
| [28,] -2.84550542 -0.76780502 1.15168793 0.311354436 | |
| [29,] 2.35995585 -0.01790055 0.03648498 -0.032804291 | |
| [30,] -0.17974128 -1.43493745 -0.75677041 0.240936580 | |
| [31,] -1.96012351 0.14141308 0.18184598 -0.336121113 | |
| [32,] -1.66566662 -0.81491072 -0.63661186 -0.013348844 | |
| [33,] -1.11208808 2.20561081 -0.85489245 -0.944789648 | |
| [34,] 2.96215223 0.59309738 0.29824930 -0.251434626 | |
| [35,] 0.22369436 -0.73477837 -0.03082616 0.469152817 | |
| [36,] 0.30864928 -0.28496113 -0.01515592 0.010228476 | |
| [37,] -0.05852787 -0.53596999 0.93038718 -0.235390872 | |
| [38,] 0.87948680 -0.56536050 -0.39660218 0.355452378 | |
| [39,] 0.85509072 -1.47698328 -1.35617705 -0.607402746 | |

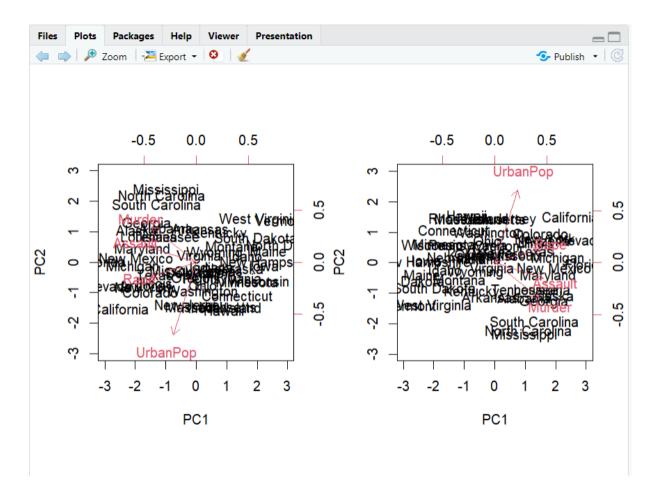
| 100,1 1.1100 | 000 E.20901001 0.09105219 0.911/05010 |
|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| [34,] 2.96215 | |
| | 436 -0.73477837 -0.03082616 0.469152817 |
| | 928 -0.28496113 -0.01515592 0.010228476 |
| | 787 -0.53596999 0.93038718 -0.235390872 |
| | 680 -0.56536050 -0.39660218 0.355452378 |
| | 072 -1.47698328 -1.35617705 -0.607402746 |
| | 986 1.91397297 -0.29751723 -0.130145378 |
| | 669 0.81506822 0.38538073 -0.108470512 |
| [42] -0 98969 | 377 0.85160534 0.18619262 0.646302674 |
| | 838 -0.40833518 -0.48712332 0.636731051 |
| | 180 -1.45671524 0.29077592 -0.081486749 |
| [45,] 2,77325 | 613 1.38819435 0.83280797 -0.143433697 |
| [46,] 0.09536 | 670 0.19772785 0.01159482 0.209246429 |
| | 6131.388194350.83280797-0.1434336976700.197727850.011594820.209246429339-0.960373940.61859067-0.218628161 |
| [48] 2 08739 | 306 1.41052627 0.10372163 0.130583080 |
| | 199 -0.60512507 -0.13746933 0.182253407 |
| | 061 0.31778662 -0.23824049 -0.164976866 |
| > pcob\$x | |
| peoper | PC1 PC2 PC3 PC4 |
| Alabama | -0.97566045 1.12200121 -0.43980366 0.154696581 |
| | -1.93053788 1.06242692 2.01950027 -0.434175454 |
| Arizona | -1.93053788 1.06242692 2.01950027 -0.434175454 -1.74544285 -0.73845954 0.05423025 -0.826264240 0.13999894 1.10854226 0.11342217 -0.180973554 -2.49861285 -1.52742672 0.59254100 -0.338559240 |
| Arkansas | 0.13999894 1.10854226 0.11342217 -0.180973554 |
| California | -2.49861285 -1.52742672 0.59254100 -0.338559240 |
| Colorado | -1.49934074 -0.97762966 1.08400162 0.001450164 |
| Connecticut | 1.34499236 -1.07798362 -0.63679250 -0.117278736 |
| Delaware | -0.04722981 -0.32208890 -0.71141032 -0.873113315 |
| Florida | -2.98275967 0.03883425 -0.57103206 -0.095317042 |
| Georgia | -1.62280742 1.26608838 -0.33901818 1.065974459 |
| Hawaii | 0.90348448 -1.55467609 0.05027151 0.893733198 |
| Idaho | 1.62331903 0.20885253 0.25719021 -0.494087852 |
| Illinois | -1.36505197 -0.67498834 -0.67068647 -0.120794916 |
| Indiana | 0.50038122 -0.15003926 0.22576277 0.420397595 |
| Iowa | 2.23099579 -0.10300828 0.16291036 0.017379470 |
| Kansas | 0.78887206 -0.26744941 0.02529648 0.204421034 |
| Kentucky | 0.74331256 0.94880748 -0.02808429 0.663817237 |
| Louisiana | -1.54909076 0.86230011 -0.77560598 0.450157791 |
| Maine | 2.37274014 0.37260865 -0.06502225 -0.327138529 |
| Maryland | -1.74564663 0.42335704 -0.15566968 -0.553450589 |
| Massachusetts | 0.48128007 -1.45967706 -0.60337172 -0.177793902 |
| Michigan | -2 08725025 -0 15383500 0 38100046 0 101343128 |

| Michigan -2.08725025 -0.15383500 0.38100046 0.101343128 | |
|----------------------------------------------------------------|---|
| Minnesota 1.67566951 -0.62590670 0.15153200 0.066640316 | |
| Mississippi -0.98647919 2.36973712 -0.73336290 0.213342049 | |
| Missouri -0.68978426 -0.26070794 0.37365033 0.223554811 | |
| Montana 1.17353751 0.53147851 0.24440796 0.122498555 | |
| Nebraska 1.25291625 -0.19200440 0.17380930 0.015733156 | |
| Nevada -2.84550542 -0.76780502 1.15168793 0.311354436 | |
| New Hampshire 2.35995585 -0.01790055 0.03648498 -0.032804291 | |
| New Jersey -0.17974128 -1.43493745 -0.75677041 0.240936580 | |
| New Mexico -1.96012351 0.14141308 0.18184598 -0.336121113 | |
| New York -1.66566662 -0.81491072 -0.63661186 -0.013348844 | |
| North Carolina -1.11208808 2.20561081 -0.85489245 -0.944789648 | |
| North Dakota 2.96215223 0.59309738 0.29824930 -0.251434626 | |
| Ohio 0.22369436 -0.73477837 -0.03082616 0.469152817 | |
| Oklahoma 0.30864928 -0.28496113 -0.01515592 0.010228476 | |
| Oregon -0.05852787 -0.53596999 0.93038718 -0.235390872 | |
| Pennsylvania 0.87948680 -0.56536050 -0.39660218 0.355452378 | |
| Rhode Island 0.85509072 -1.47698328 -1.35617705 -0.607402746 | |
| South Carolina -1.30744986 | |
| South Dakota | |
| Tennessee -0.98969377 0.85160534 0.18619262 0.646302674 | |
| Texas -1.34151838 -0.40833518 -0.48712332 0.636731051 | |
| Utah 0.54503180 -1.45671524 0.29077592 -0.081486749 | |
| Vermont 2.77325613 1.38819435 0.83280797 -0.143433697 | |
| Virginia 0.09536670 0.19772785 0.01159482 0.209246429 | |
| Washington 0.21472339 -0.96037394 0.61859067 -0.218628161 | |
| West Virginia 2.08739306 1.41052627 0.10372163 0.130583080 | |
| Wisconsin 2.05881199 -0.60512507 -0.13746933 0.182253407 | |
| Wyoming 0.62310061 0.31778662 -0.23824049 -0.164976866 | |
| > nomit <- 20 | |
| > set.seed(15) | |
| > ina <- sample(seq(50), nomit) | |
| <pre>> inb <- sample(1:4, nomit, replace = TRUE)</pre> | |
| > Xna <- X | |
| > index.na <- cbind(ina, inb) | |
| > Xna[index.na] <- NA | |
| <pre>> fit.svd <- function(X, M = 1) {</pre> | |
| + $svdob < - svd(x)$ | |
| + with(svdob, | |
| + $u[, 1:M, drop = FALSE] \%\%$ | |
| + $(d[:M] * t(V[, 1:M, drop = FALSE]))$ | - |
| | |

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```
+
           u[, 1:M, drop = FALSE] %*%
+
             (d[1:M] * t(v[, 1:M, drop = FALSE]))
+
+
+ }
    )
>
> Xhat <- Xna
> xbar <- colMeans(Xna, na.rm = TRUE)</pre>
> Xhat[index.na] <- xbar[inb]</pre>
> thresh <- 1e-7
> rel_err <- 1
> iter <- 0
> ismiss <- is.na(Xna)</pre>
> mssold <- mean((scale(Xna, xbar, FALSE)[!ismiss])^2)
> msso <- mean(Xna[!ismiss]^2)</pre>
> while(rel_err > thresh) {
    iter <- iter + 1
+
     # Step 2(a)
+
    Xapp <- fit.svd(Xhat, M = 1)</pre>
     # Step 2(b)
+
    Xhat[ismiss] <- Xapp[ismiss]</pre>
+
    # step 2(c)
mss <- mean(((Xna - Xapp)[!ismiss])^2)
rel_err <- (mssold - mss) / mss0</pre>
+
+
+
    +
+
+
+ }
Iter: 1 MSS: 0.3821695 Rel. Err: 0.6194004
Iter: 2 MSS: 0.3705046 Rel. Err: 0.01161265
Iter: 3 MSS: 0.3692779 Rel. Err: 0.001221144
Iter: 4 MSS: 0.3691229 Rel. Err: 0.0001543015
Iter: 5 MSS: 0.3691008 Rel. Err: 2.199233e-05
Iter: 6 MSS: 0.3690974 Rel. Err: 3.376005e-06
Iter: 7 MSS: 0.3690969 Rel. Err: 5.465067e-07
Iter: 8 MSS: 0.3690968 Rel. Err: 9.253082e-08
> cor(Xapp[ismiss], X[ismiss])
[1] 0.6535043
> |
```





Based on the analysis, it was found that the first seven principal components together explain only 40% of the variance in the data. While each of the first seven components explains a significant amount of variance, there is a significant decrease in the variance explained by further principal components. This suggests that examining more than seven or so principal components may not be very beneficial.

These findings are in contrast to the results obtained when hierarchical clustering was performed on the full data set. It is possible that clustering on the first few principal component score vectors could yield better results than clustering on the full data. This step can be viewed as a way to denoise the data. Another option is to perform K-means clustering on the first few principal component score vectors instead of the full data set.

2)

Code:-

set.seed(2)

x <- matrix(rnorm(50 * 2), ncol = 2) x[1:25, 1] <- x[1:25, 1] + 3 x[1:25, 2] <- x[1:25, 2] - 4 km.out <- kmeans(x, 2, nstart = 20) km.out\$cluster

 $plot(x, col = (km.out\cluster + 1),$

```
main = "K-Means Clustering Results with K = 2",
xlab = "", ylab = "", pch = 20, cex = 2)
set.seed(4)
km.out <- kmeans(x, 3, nstart = 20)
km.out
plot(x, col = (km.out cluster + 1),
main = "K-Means Clustering Results with K = 3",
xlab = "", ylab = "", pch = 20, cex = 2)
set.seed(4)
km.out <- kmeans(x, 3, nstart = 1)
km.out$tot.withinss
km.out <- kmeans(x, 3, nstart = 20)
km.out$tot.withinss
hc.complete <- hclust(dist(x), method = "complete")</pre>
hc.average <- hclust(dist(x), method = "average")</pre>
hc.single <- hclust(dist(x), method = "single")</pre>
par(mfrow = c(1, 3))
plot(hc.complete, main = "Complete Linkage",
xlab = "", sub = "", cex = .9)
plot(hc.average, main = "Average Linkage",
xlab = "", sub = "", cex = .9)
plot(hc.single, main = "Single Linkage",
xlab = "", sub = "", cex = .9)
cutree(hc.complete, 2)
cutree(hc.average, 2)
cutree(hc.single, 2)
cutree(hc.single, 4)
xsc <- scale(x)</pre>
plot(hclust(dist(xsc), method = "complete"),
main = "Hierarchical Clustering with Scaled Features")
```

x <- matrix(rnorm(30 * 3), ncol = 3)

 $dd \leq as.dist(1 - cor(t(x)))$

plot(hclust(dd, method = "complete"),

main = "Complete Linkage with Correlation-Based Distance",

xlab = "", sub = "")

Output:-

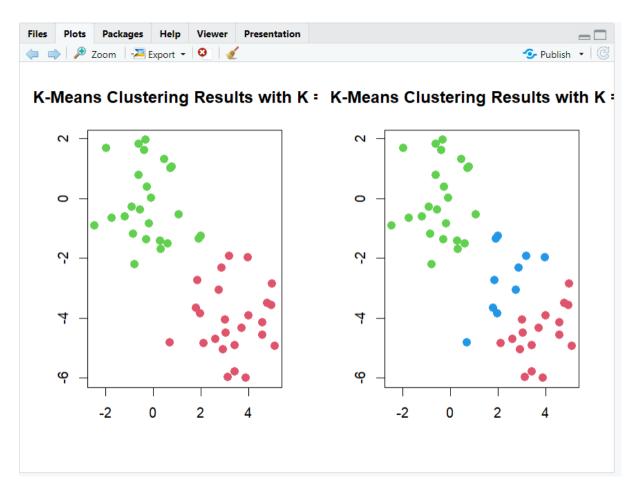
```
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> set.seed(2)
> x <- matrix(rnorm(50 * 2), ncol = 2)
> x[1:25, 1] <- x[1:25, 1] + 3
> x[1:25, 2] <- x[1:25, 2] - 4
> km.out <- kmeans(x, 2, nstart = 20)</pre>
> km.out$cluster
 [46] 2 2 2 2 2 2
> plot(x, col = (km.out$cluster + 1),
+ main = "K-Means Clustering Results with K = 2",
+ xlab = "", ylab = "", pch = 20, cex = 2)
> set.seed(4)
> km.out <- kmeans(x, 3, nstart = 20)
> km.out
K-means clustering with 3 clusters of sizes 17, 23, 10
Cluster means:
        [,1]
                     [,2]
1 3.7789567 -4.56200798
2 -0.3820397 -0.08740753
3 2.3001545 -2.69622023
Clustering vector:
Within cluster sum of squares by cluster:
[1] 25.74089 52.67700 19.56137
 (between_SS / total_SS = 79.3 %)
Available components:
[1] "cluster"
[7] "size"
                    "centers"
                                    "totss"
                                                                   "tot.withinss" "betweenss"
                                                    "withinss"
                   "iter"
                                    "ifault"
> plot(x, col = (km.out$cluster + 1),
       main = "K-Means Clustering Results with K = 3",
xlab = "", ylab = "", pch = 20, cex = 2)
+
> set.seed(4)
> km.out <- kmeans(x, 3, nstart = 1)
> km.out$tot.withinss
[1] 104.3319
```

```
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🗣 R 4.2.2 · ~/ 🗇
       main = "K-Means Clustering Results with K = 3",
xlab = "", ylab = "", pch = 20, cex = 2)
                                                                                                              *
+
+
> set.seed(4)
> km.out <- kmeans(x, 3, nstart = 1)</pre>
> km.out$tot.withinss
[1] 104.3319
> km.out <- kmeans(x, 3, nstart = 20)
> km.out$tot.withinss
[1] 97.97927
> hc.complete <- hclust(dist(x), method = "complete")
> hc.average <- hclust(dist(x), method = "average")
> hc.single <- hclust(dist(x), method = "single")</pre>
> par(mfrow = c(1, 3))
> par(infrow = C(1, 3))
> plot(hc.complete, main = "Complete Linkage",
+ xlab = "", sub = "", cex = .9)
> plot(hc.average, main = "Average Linkage",
+ xlab = "", sub = "", cex = .9)
> plot(hc.single, main = "Single Linkage",
+ xlab = "", sub = "", cex = .9)
> cutree(hc.complete, 2)
[46] 1 2 2 2 2
> cutree(hc.single, 2)
 [46] 1 1 1 1 1
> cutree(hc.single, 4)
> xsc <- scale(x)</pre>
> plot(hclust(dist(xsc), method = "complete"),
      main = "Hierarchical Clustering with Scaled Features")
> x <- matrix(rnorm(30 * 3), ncol = 3)
> dd <- as.dist(1 - cor(t(x)))</pre>
> plot(hclust(dd, method = "complete"),
+ main = "Complete Linkage with Correlation-Based Distance",
+ xlab = "", sub = "")
> |
```

Plots:-



Findings:-

The code uses the elbow method to determine the optimal number of clusters for the USArrests dataset. This involves plotting the WSS against the number of clusters and selecting the point where the decrease in WSS begins to level off. In this case, the elbow occurs at k=3, so k-means clustering is performed with centers=3. The resulting cluster centers show the average values of each variable for each cluster.

3)

Code:-

library(ISLR2) nci.labs <- NCI60\$labs nci.data <- NCI60\$data dim(nci.data) nci.labs[1:4] table(nci.labs) pr.out <- prcomp(nci.data, scale = TRUE)

 $Cols <- \ function(vec) \ \{$

cols <- rainbow(length(unique(vec)))

```
return(cols[as.numeric(as.factor(vec))])
}
par(mfrow = c(1, 2))
plot(pr.out$x[, 1:2], col = Cols(nci.labs), pch = 19,
xlab = "Z1", ylab = "Z2")
plot(pr.out$x[, c(1, 3)], col = Cols(nci.labs), pch = 19,
xlab = "Z1", ylab = "Z3")
summary(pr.out)
plot(pr.out)
pve <- 100 * pr.out$sdev^2 / sum(pr.out$sdev^2)
par(mfrow = c(1, 2))
plot(pve, type = "o", ylab = "PVE",
xlab = "Principal Component", col = "blue")
plot(cumsum(pve), type = "o", ylab = "Cumulative PVE",
xlab = "Principal Component", col = "brown3")
sd.data <- scale(nci.data)</pre>
par(mfrow = c(1, 3))
data.dist <- dist(sd.data)</pre>
plot(hclust(data.dist), xlab = "", sub = "", ylab = "",
labels = nci.labs, main = "Complete Linkage")
plot(hclust(data.dist, method = "average"),
labels = nci.labs, main = "Average Linkage",
xlab = "", sub = "", ylab = "")
plot(hclust(data.dist, method = "single"),
labels = nci.labs, main = "Single Linkage",
xlab = "", sub = "", ylab = "")
hc.out <- hclust(dist(sd.data))</pre>
hc.clusters <- cutree(hc.out, 4)
table(hc.clusters, nci.labs)
par(mfrow = c(1, 1))
```

plot(hc.out, labels = nci.labs)

abline(h = 139, col = "red")

set.seed(2)

km.out <- kmeans(sd.data, 4, nstart = 20)

km.clusters <- km.out\$cluster

table(km.clusters, hc.clusters)

hc.out <- hclust(dist(pr.out\$x[, 1:5]))</pre>

plot(hc.out, labels = nci.labs,

main = "Hier. Clust. on First Five Score Vectors")

table(cutree(hc.out, 4), nci.labs)

Output:-

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|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| R 4.2.2 · ~/ [→] | |
| > library(ISLR2) | |
| > nci.labs <- NCI60\$labs | |
| > nci.data <- NCI60\$data | |
| > dim(nci.data) | |
| [1] 64 6830 > nci.labs[1:4] | |
| [1] "CNS" "CNS" "CNS" "RENAL" | |
| > table(nci, labs) | |
| nci.labs | |
| BREAST CNS COLON K562A-repro K562B-repro LEUKEMIA MCF7A-repro | |
| 7 5 7 1 1 6 1 | |
| MCF7D-repro MELANOMA NSCLC OVARIAN PROSTATE RENAL UNKNOWN | |
| | |
| <pre>> pr.out <- prcomp(nci.data, scale = TRUE) > Cols <- function(vec) {</pre> | |
| + cols <- rainbow(length(unique(vec))) | |
| + return(cols[as.numeric(as.factor(vec))]) | |
| + } | |
| > par(mfrow = c(1, 2)) | |
| <pre>> plot(pr.out\$x[, 1:2], col = Cols(nci.labs), pch = 19,</pre> | |
| + $x lab = "Z1", y lab = "Z2")$ | |
| <pre>> plot(pr.out\$x[, c(1, 3)], col = Cols(nci.labs), pch = 19, + xlab = "Z1", ylab = "Z3")</pre> | |
| + x ab = 21, y ab = 23 > summary(pr.out) | |
| Importance of components: | |
| PC1 PC2 PC3 PC4 PC5 PC6 PC7 PC8 | |
| Standard deviation 27.8535 21.48136 19.82046 17.03256 15.97181 15.72108 14.47145 13.54427 | |
| Proportion of Variance 0.1136 0.06756 0.05752 0.04248 0.03735 0.03619 0.03066 0.02686 | |
| Cumulative Proportion 0.1136 0.18115 0.23867 0.28115 0.31850 0.35468 0.38534 0.41220 | _ |
| PC9 PC10 PC11 PC12 PC13 PC14 PC15 PC10 | |
| Standard deviation 13.14400 12.73860 12.68672 12.15769 11.83019 11.62554 11.43779 11.00052 Proportion of Variance 0.02529 0.02376 0.02357 0.02164 0.02049 0.01979 0.01915 0.01772 | |
| Cumulative Proportion 0.43750 0.46126 0.48482 0.50646 0.52695 0.54674 0.56590 0.5836 | |
| PC17 PC18 PC19 PC20 PC21 PC22 PC23 PC24 | |
| Standard deviation 10.65666 10.48880 10.43518 10.3219 10.14608 10.0544 9.90265 9.64766 | |
| Proportion of Variance 0.01663 0.01611 0.01594 0.0156 0.01507 0.0148 0.01436 0.01363 | |
| Cumulative Proportion 0.60024 0.61635 0.63229 0.6479 0.66296 0.6778 0.69212 0.70575 | |
| PC25 PC26 PC27 PC28 PC29 PC30 PC31 PC32 PC33 | |
| Standard deviation 9.50764 9.33253 9.27320 9.0900 8.98117 8.75003 8.59962 8.44738 8.37305 | |
| Proportion of Variance 0.01324 0.01275 0.01259 0.0121 0.01181 0.01121 0.01083 0.01045 0.01026 Cumulative Proportion 0.71899 0.73174 0.74433 0.7564 0.76824 0.77945 0.79027 0.80072 0.81099 | - |
| Cullin active Proportion 0.71699 0.751/4 0.74455 0.7504 0.70624 0.77945 0.79027 0.800/2 0.81099 | |

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|------------------------------------------|------------|---------------------------|-------------|-----------|-----------|-----------|-----------|----------|--|
| K 4.2.2 · ~/ W | 0.014 | DC35 DC | 26 8627 | 0630 | 0.020 | 0.040 | 0.041 | 0643 | |
| e | PC34 | | 36 PC37 | PC38 | | PC40 | PC41 | PC42 | |
| Standard deviation | | 8.15731 7.974 | | | | | | | |
| Proportion of Variance | | | | | | | | | |
| Cumulative Proportion | | | | | | | | | |
| | PC43 | PC44 PC4 | | PC47 | PC48 | PC49 | PC50 | PC51 | |
| Standard deviation | | 7.0131 6.9583 | | | | | | | |
| Proportion of Variance | | | | | | | | | |
| Cumulative Proportion | | | | | | | | | |
| | PC52 | | 54 PC55 | PC56 | PC57 | PC58 | PC59 | PC60 | |
| Standard deviation | | 6.06706 5.918 | | | | | | | |
| Proportion of Variance | | | | | | | | | |
| Cumulative Proportion | | | | | 0.98161 | 0.9857 (|).98940 (| 0.99262 | |
| | PC61 | | 63 PC | | | | | | |
| Standard deviation | | 4.08212 4.041 | | | | | | | |
| Proportion of Variance | | | | | | | | | |
| Cumulative Proportion | 0.99517 | 0.99761 1.000 | 000 1.000e+ | 00 | | | | | |
| > plot(pr.out) | | | | | | | | | |
| > pve <- 100 * pr.out\$ | sdev^2 / | <pre>sum(pr.out\$sc</pre> | lev^2) | | | | | | |
| > par(mfrow = $c(1, 2)$) | | | | | | | | | |
| <pre>> plot(pve, type = "o"</pre> | | | | | | | | | |
| + xlab = "Principa | | | | | | | | | |
| <pre>> plot(cumsum(pve), typ</pre> | | | | ', | | | | | |
| + xlab = "Principa | | nent", col = " | 'brown3") | | | | | | |
| > sd.data <- scale(nci | data) | | | | | | | | |
| > par(mfrow = $c(1, 3)$) | | | | | | | | | |
| <pre>> data.dist <- dist(sd)</pre> | | | | | | | | | |
| > plot(hclust(data.dist | | | | · · · | | | | | |
| + labels = nci.lal | | | | | | | | | |
| > plot(hclust(data.dist | | | | | | | | | |
| + labels = nci.la | os, main | = "Average Li | nkage", | | | | | | |
| + xlab = "", sub = | | | | | | | | | |
| <pre>> plot(hclust(data.dist</pre> | | | | | | | | | |
| + labels = nci.lal | os, mair | ı = "Single Li | nkage", | | | | | | |
| + xlab = "", sub = | | | | | | | | | |
| > hc.out <- hclust(dist | | 2 C | | | | | | | |
| > hc.clusters <- cutree | | 4) | | | | | | | |
| > table(hc.clusters, no | i.labs) | | | | | | | | |
| nci.labs | | | | | | | | | |
| hc.clusters BREAST CNS | | 62A-repro K56 | 2B-repro L | EUKEMIA I | MCF7A-rep | pro MCF7[| D-repro I | MELANOMA | |
| 1 2 3 | 2 | 0 | 0 | 0 | | 0 | 0 | 8 | |
| 2 3 2 | 0 | 0 | 0 | 0 | | 0 | Ō | 0 | |

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                  0
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> par(mfrow = c(1, 1))
> plot(hc.out, labels = nci.labs)
> abline(h = 139, col = "red")
> hc.out
Call:
hclust(d = dist(sd.data))
Cluster method : complete
Distance : euclidean
Number of objects: 64
> set.seed(2)
> km.out <- kmeans(sd.data, 4, nstart = 20)</pre>
> km.clusters <- km.out$cluster</pre>
> table(km.clusters, hc.clusters)
           hc.clusters
km.clusters 1 2 3 4
1 11 0 0 9
           2 20 7 0 0
3 9 0 0 0
           4 0 0 8 0
> hc.out <- hclust(dist(pr.out$x[, 1:5]))</pre>
> plot(hc.out, labels = nci.labs,
+ main = "Hier. Clust. on First Five Score Vectors")
> table(cutree(hc.out, 4), nci.labs)
   nci.labs
    BREAST CNS COLON K562A-repro K562B-repro LEUKEMIA MCF7A-repro MCF7D-repro MELANOMA NSCLC
             2
          0
                                                  0
                                                                           0
  1
                      7
                                    0
                                                             2
                                                                                         0
                                                                                                    1
                                                                                                           8
                                                                                                    7
  2
                      0
          5
               3
                                    0
                                                   0
                                                             0
                                                                           0
                                                                                         0
                                                                                                           1
  3
          0
               0
                                                                                                    0
                                                                                                           0
                      0
                                    1
                                                   1
                                                             4
                                                                           0
                                                                                         0
  4
          2
               0
                      0
                                    0
                                                   0
                                                             0
                                                                           1
                                                                                         1
                                                                                                    0
                                                                                                           0
  nci.labs
    OVARIAN PROSTATE RENAL UNKNOWN
                      2
                            7
                                       0
  1
          5
  2
                      0
                             2
           1
                                       1
            0
                             0
  3
                      0
                                       0
  4
           0
                      0
                                       0
                             0
>
```

Plots:-

