## FINDINGS:

The code performs PCA on the USArrests dataset using the prcomp() function with scale = TRUE to standardize the variables. The summary() function displays information on the variance explained by each principal component, as well as the loadings of the variables on each component.

The standard deviations of the four principal components are displayed in the first row of the summary, with the first principal component explaining the most variance ( 1.57 units) and the fourth component explaining the least ( 0.56 units). Together, the four components explain all the variance in the data.

The rotation matrix displayed in the second row of the summary gives the loadings of each variable on each component. For example, the loading of Murder on PC1 is -0.5359 , indicating that states with high murder rates tend to have low scores on PC1. The loading of UrbanPop on PC2 is -0.8728 , meaning that states with high percentages of urban population tend to have low scores on PC2.

The interpretation of each principal component depends on the loadings of the variables. In this case, PC1 can be interpreted as a measure of overall crime rate, as it is positively correlated with all variables in the dataset. PC2 can be interpreted as a measure of urbanization, as it is negatively correlated with UrbanPop. PC3 is a measure of sexual assault, as it is strongly positively correlated with Rape. PC4 is a measure of the difference between violent and non-violent crimes, as it is positively correlated with Murder and negatively correlated with Assault

## CODE:

```
data(USArrests)
states <- row.names(USArrests)
names(USArrests)
apply(USArrests, 2, mean)
apply(USArrests, 2, var)
pr.out <- prcomp(USArrests, scale = TRUE)
```

```
names(pr.out)
pr.out$center
pr.out$scale
pr.out$rotation
dim(pr.out$x)
biplot(pr.out, scale = 0)
pr.out$rotation = -pr.out$rotation
pr.out$x = -pr.out$x
biplot(pr.out, scale = 0)
pr.out$sdev
pr.var<- pr.out$sdev^2
pve <- pr.var / sum(pr.var)
par(mfrow = c(1, 2))
plot(pve, xlab = "Principal Component",
    ylab = "Proportion of Variance Explained", ylim =c(0,1),
    type = "b")
plot(cumsum(pve), xlab = "Principal Component",
    ylab = "Cumulative Proportion of Variance Explained",
    ylim = c(0, 1), type = "b")
a<-c(1, 2, 8, -3)
cumsum(a)
X <- data.matrix(scale(USArrests))
pcob <- prcomp(X)
summary(pcob)
sX <- svd(X)
names(sX)
round(sX$v, 3)
pcob$rotation
```

```
t(sX$d * t(sX$u))
pcob$x
nomit <- 20
set.seed(15)
ina <- sample(seq(50), nomit)
inb <- sample(1:4, nomit, replace = TRUE)
Xna <- X
index.na <- cbind(ina, inb)
Xna[index.na] <- NA
fit.svd <- function(X,M=1) {
    svdob <- svd(X)
    with(svdob,
        u[, 1:M, drop = FALSE] %*%
        (d[1:M] * t(v[, 1:M, drop = FALSE] ) )
    )
}
Xhat <- Xna
xbar <- colMeans(Xna, na.rm = TRUE)
Xhat[index.na] <- xbar[inb]
thresh <- 1e-7
rel_err <- 1
iter <- 0
ismiss <- is.na(Xna)
mssold <- mean((scale(Xna, xbar, FALSE)[!ismiss])^2)
mss0<- mean(Xna[!ismiss]^2)
```

while(rel_err > thresh) \{

```
iter <- iter + 1
Xapp <- fit.svd(Xhat, M=1)
Xhat[ismiss] <- Xapp[ismiss]
mss<- mean(((Xna- Xapp)[!ismiss])^2)
rel_err <- (mssold - mss) / mss0
mssold <- mss
cat("Iter:", iter, "MSS:", mss,
    "Rel. Err:", rel_err, "\n")
}
cor(Xapp[ismiss], X[ismiss])
```



```
<<-2
R R 4.2.2 ~ ~/
> pr.out$sdev
[1] 1.5748783 0.9948694 0.5971291 0.4164494
> pr.var <- pr.out$sdev^2
> pr.var
[1] 2.4802416 0.9897652 0.3565632 0.1734301
> pve <- pr.var / sum(pr.var)
> pve
[1] 0.62006039 0.24744129 0.08914080 0.04335752
> par(mfrow = c(1, 2))
> plot(pve, xlab = "Principal Component",
+ ylab = "Proportion of Variance Explained", ylim = c(0, 1),
+ type = "b")
> plot(cumsum(pve), xlab = "Principal Component"
+ y`ab = "Cumulative Proportion of Variance Explained",
+ ylim = c(0, 1), type = "b")
> a <- c(1, 2, 8, -3)
> cumsum(a)
[1] 1 3 11 8
> X <- data.matrix(scale(USArrests))
> pcob <- prcomp(X)
> summary(pcob)
Importance of components:
                    PC1 PC2 
Standard deviation 1.5749 0.9949 0.59713 0.41645
Proportion of Variance 0.6201 0.2474 0.08914 0.04336
Cumulative Proportion 0.6201 0.8675 0.95664 1.00000
> sX <- svd(X)
> names (sX)
[1] "d" "u" "v"
> round(sX$v, 3)
[,1] [,2] [,3] [,4]
[1,] -0.536 0.418
[2,] -0.583 0.188 -0.268 -0.743
[3,] -0.278 -0.873 -0.378 0.134
[4,] -0.543 -0.167 0.818
> pcob$rotation
    PC1 PC2 PC3 PC4
Murder -0.5358995 0.4181809 -0.3412327 0.64922780
Assau7t -0.5831836 0.1879856 -0.2681484 -0.74340748
UrbanPop -0.2781909 -0.8728062 -0.3780158 0.13387773
Rane -n 5424271 -n 1672186 n 8177777a n n8anว43)
```

| Console | Terminal $\times$ | Background Jobs |  |  | - |
| :---: | :---: | :---: | :---: | :---: | :---: |
| R R 4.2.2 $\sim / \sim$ |  |  |  |  |  |
| $>\mathrm{t}(\mathrm{sX} \mathrm{\$ d}$ * $\mathrm{t}(\mathrm{s} \times \$ \mathrm{u})) \mathrm{l}$ |  |  |  |  |  |
|  | [,1] | [,2] | [,3] | [,4] |  |
| [1, ] | -0.97566045 | 1.12200121 | -0.43980366 | 0.154696581 |  |
| [2,] | -1.93053788 | 1.06242692 | 2.01950027 | -0.434175454 |  |
| [3, ] | -1.74544285 | -0.73845954 | 0.05423025 | -0.826264240 |  |
| [4,] | 0.13999894 | 1.10854226 | 0.11342217 | -0.180973554 |  |
| [5,] | -2.49861285 | -1.52742672 | 0.59254100 | -0.338559240 |  |
| [6, ] | -1.49934074 | -0.97762966 | 1.08400162 | 0.001450164 |  |
| [7, ] | 1. 34499236 | -1.07798362 | -0.63679250 | -0.117278736 |  |
| [8,] | -0.04722981 | -0.32208890 | -0.71141032 | -0.873113315 |  |
| [9,] | -2.98275967 | 0.03883425 | -0.57103206 | -0.095317042 |  |
| [10,] | -1.62280742 | 1.26608838 | -0.33901818 | 1.065974459 |  |
| [11,] | 0.90348448 | -1.55467609 | 0.05027151 | 0.893733198 |  |
| [12,] | 1.62331903 | 0.20885253 | 0.25719021 | -0.494087852 |  |
| [13,] | -1.36505197 | -0.67498834 | -0.67068647 | -0.120794916 |  |
| [14, ] | 0.50038122 | -0.15003926 | 0.22576277 | 0.420397595 |  |
| [15,] | 2.23099579 | -0.10300828 | 0.16291036 | 0.017379470 |  |
| [16, ] | 0.78887206 | -0.26744941 | 0.02529648 | 0.204421034 |  |
| [17,] | 0.74331256 | 6.94880748 | -0.02808429 | 0.663817237 |  |
| [18, ] | -1.54909076 | 6.86230011 | -0.77560598 | 0.450157791 |  |
| [19,] | 2.37274014 | 40.37260865 | -0.06502225 | -0.327138529 |  |
| [20,] | -1.74564663 | 0.42335704 | -0.15566968 | -0.553450589 |  |
| [21,] | 0.48128007 | -1.45967706 | -0.60337172 | -0.177793902 |  |
| [22,] | -2.08725025 | -0.15383500 | 0.38100046 | 0.101343128 |  |
| [23,] | 1.67566951 | -0.62590670 | 0.15153200 | 0.066640316 |  |
| [24,] | -0.98647919 | 2.36973712 | -0.73336290 | 0.213342049 |  |
| [25,] | -0.68978426 | -0.26070794 | 0.37365033 | 0.223554811 |  |
| [26, ] | 1.17353751 | 0.53147851 | 0.24440796 | 0.122498555 |  |
| [27, ] | 1.25291625 | -0.19200440 | 0.17380930 | 0.015733156 |  |
| [28, ] | -2.84550542 | -0.76780502 | 1.15168793 | 0.311354436 |  |
| [29,] | 2.35995585 | -0.01790055 | 0.03648498 | -0.032804291 |  |
| [30,] | -0.17974128 | -1.43493745 | -0.75677041 | 0.240936580 |  |
| [31,] | -1.96012351 | 0.14141308 | 0.18184598 | -0.336121113 |  |
| [32,] | -1.66566662 | -0.81491072 | -0.63661186 | -0.013348844 |  |
| [33,] | -1.11208808 | 2.20561081 | -0.85489245 | -0.944789648 |  |
| [34,] | 2.96215223 | 0.59309738 | 0.29824930 | -0.251434626 |  |
| [35,] | 0.22369436 | -0.73477837 | -0.03082616 | 0.469152817 |  |
| [36,] | 0.30864928 | -0.28496113 | -0.01515592 | 0.010228476 |  |
| [37, ] | -0.05852787 | -0.53596999 | 0.93038718 | -0.235390872 |  |
| [38,] | 0.87948680 | -0.56536050 | -0.39660218 | 0.355452378 |  |
| 「39.1 | 0.85509072 | -1.47698328 | -1.35617705 | -0.607402746 |  |




```
R R 4.2.2 ~/ 
+ u[, 1:M, drop = FALSE] %*%
+ (d[1:M] * t(v[, 1:M, drop = FALSE]))
    )
+ }
Xhat <- Xna
xbar <- colMeans(Xna, na.rm = TRUE)
Xhat[index.na] <- xbar[inb]
thresh <- 1e-7
rel_err <- 1
> iter <- 0
> ismiss <- is.na(Xna)
> mssold <- mean((scale(Xna, xbar, FALSE)[!ismiss])^2)
> mss0 <- mean(Xna[!ismiss]^2)
> while(rel_err > thresh) {
    iter <- iter + 1
        # Step 2(a)
        Xapp <- fit.svd(Xhat, M = 1)
        # Step 2(b)
        Xhat[ismiss] <- Xapp[ismiss]
        # Step 2(c)
        mss <- mean(((Xna - Xapp)[!ismiss])^2)
        rel_err <- (mssold - mss) / mss0
        mssold <- mss
        cat("Iter:", iter, "MSs:", mss,
            "Rel. Err:", rel_err, "\n")
+ }
Iter: 1 MSS: 0.3821695 Re1. Err: 0.6194004
Iter: 2 MSS: 0.3705046 Rel. Err: 0.01161265
Iter: 3 MSS: 0.3692779 Re1. Err: 0.001221144
Iter: 4 MSS: 0.3691229 Re1. Err: 0.0001543015
Iter: 5 MSS: 0.3691008 Re1. Err: 2.199233e-05
Iter: 6 MSS: 0.3690974 Re7. Err: 3.376005e-06
Iter: 7 MSS: 0.3690969 Re1. Err: 5.465067e-07
Iter: 8 MSS: 0.3690968 Re1. Err: 9.253082e-08
>
> cor(Xapp[ismiss], X[ismiss])
[1] 0.6535043
>
```

```Files Plots Packages Help Viewer Presentation




Based on the analysis, it was found that the first seven principal components together explain only \(40 \%\) of the variance in the data. While each of the first seven components explains a significant amount of variance, there is a significant decrease in the variance explained by further principal components. This suggests that examining more than seven or so principal components may not be very beneficial.

These findings are in contrast to the results obtained when hierarchical clustering was performed on the full data set. It is possible that clustering on the first few principal component score vectors could yield better results than clustering on the full data. This step can be viewed as a way to denoise the data. Another option is to perform Kmeans clustering on the first few principal component score vectors instead of the full data set.
2)

Code:-
set.seed(2)
\(\mathrm{x}<-\operatorname{matrix}(\operatorname{rnorm}(50 * 2)\), ncol =2)
\(x[1: 25,1]<-x[1: 25,1]+3\)
\(x[1: 25,2]<-x[1: 25,2]-4\)
km.out <- kmeans(x, 2, nstart \(=20\) )
km.out\$cluster
\(\operatorname{plot}(\mathrm{x}, \operatorname{col}=(\mathrm{km}\). out\$cluster +1\()\),
main \(=\) "K-Means Clustering Results with \(\mathrm{K}=2\) ",
xlab = "", ylab = " ", pch = 20, cex = 2)
set.seed(4)
km.out \(<-\operatorname{kmeans}(\mathrm{x}, 3\), nstart \(=20)\)
km.out
\(\operatorname{plot}(x, \operatorname{col}=(\mathrm{km}\). out\$cluster +1\()\),
main \(=\) "K-Means Clustering Results with \(K=3\) ",
xlab = "", ylab = "", pch = 20, cex = 2)
set.seed(4)
km.out <- \(\operatorname{kmeans}(\mathrm{x}, 3\), nstart \(=1)\)
km.out\$tot.withinss
km.out <- kmeans(x, 3, nstart \(=20\) )
km.out\$tot.withinss
hc.complete <- hclust(dist(x), method = "complete")
hc.average <- hclust(dist( x ), method = "average")
hc.single <- hclust(dist(x), method = "single")
\(\operatorname{par}(\) mfrow \(=c(1,3))\)
plot(hc.complete, main = "Complete Linkage",
xlab = "", sub = "", cex = .9)
plot(hc.average, main = "Average Linkage",
xlab = "", sub = "", cex = .9)
plot(hc.single, main = "Single Linkage",
xlab = "", sub = "", cex = .9)
cutree(hc.complete, 2)
cutree(hc.average, 2)
cutree(hc.single, 2)
cutree(hc.single, 4)
xsc <- scale ( x )
plot(hclust(dist(xsc), method = "complete"),
main \(=\) "Hierarchical Clustering with Scaled Features")
\(\mathrm{x}<-\operatorname{matrix}(\operatorname{rnorm}(30 * 3)\), ncol \(=3\) )
dd \(<-\operatorname{as.dist}(1-\operatorname{cor}(\mathrm{t}(\mathrm{x})))\)
plot(hclust(dd, method = "complete"),
main = "Complete Linkage with Correlation-Based Distance",
xlab = "", sub = "")

Output:-
```

Console Terminal }\times\mathrm{ Background Jobs }\times~~
R R4.2.2 . ~/ }
> set.seed(2)
> x <- matrix(rnorm(50 * 2), ncol = 2)
> x[1:25, 1] <- x[1:25, 1] + 3
> x[1:25, 2] <- x[1:25, 2] - 4
>km.out <- kmeans (x, 2, nstart = 20)
> km.out$cluster
[1] 1 11 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
[46] 2 2 2 2 2 2
> plot (x, col = (km.out$cluster + 1),

+ main = "K-Means Clustering Results with K = 2",
+ x`ab = "", y`ab = "", pch = 20, cex = 2)
> set.seed(4)
> km. out <- kmeans(x, 3, nstart = 20)
>km.out
K-means clustering with 3 clusters of sizes 17, 23, 10
Cluster means:
[,1] [,2]
1 3.7789567 -4.56200798
2 -0.3820397-0.08740753
3 2.3001545 -2.69622023
Clustering vector:

```

```

[46] 3 2 2 2 2
Within cluster sum of squares by cluster:
[1] 25.74089 52.67700 19.56137
(between_SS / total_SS = 79.3 %)
Available components:

```
```

[1] "cluster" "centers" "totss" "withinss" "tot.withinss" "betweenss"

```
[1] "cluster" "centers" "totss" "withinss" "tot.withinss" "betweenss"
[7] "size" "iter" "ifault"
[7] "size" "iter" "ifault"
> plot (x, col = (km.out$cluster + 1),
> plot (x, col = (km.out$cluster + 1),
+ main = "K-Means Clustering Results with K = 3",
+ main = "K-Means Clustering Results with K = 3",
+ xlab = "", ylab = "", pch = 20, cex = 2)
+ xlab = "", ylab = "", pch = 20, cex = 2)
> set.seed(4)
> set.seed(4)
> km.out <- kmeans(x, 3, nstart = 1)
> km.out <- kmeans(x, 3, nstart = 1)
> km.out$tot.withinss
> km.out$tot.withinss
「11 104.3319
```

「11 104.3319

```
```

Console Terminal × Background Jobs
\squareロ
R R4.2.2, ~/

+ main = "K-Means Clustering Results with K = 3",
+ xlab = "", y`ab = "", pch = 20, cex = 2)
> set.seed(4)
> km.out <- kmeans(x, 3, nstart = 1)
> km.out$tot.withinss
[1] 104.3319
> km.out <- kmeans(x, 3, nstart = 20)
> km.out$tot.withinss
[1] 97.97927
> hc.complete <- hclust(dist(x), method = "complete")
> hc.average <- hclust(dist(x), method = "average")
> hc.single <- hclust(dist(x), method = "single")
> par(mfrow = c(1, 3))
> plot(hc.complete, main = "Complete Linkage",
+ x7ab = "", sub = "", cex = .9)
> plot(hc.average, main = "Average Linkage",
+ xlab = "", sub = "", cex = .9)
> plot(hc.single, main = "Single Linkage",
+ xlab = "", sub = "", cex = .9)
> cutree(hc.complete, 2)

```

```

[46] 2 2 2 2 2
> cutree(hc.average, 2)

```

```

[46] 1 2 2 2 2
> cutree(hc.single, 2)

```

```

[46] 1}1011\mp@code{1}
> cutree(hc.single, 4)

```

```

[46] 3 3 3 3 3
> xsc <- scale(x)
> plot(hclust(dist(xsc), method = "complete"),

+ main = "Hierarchical Clustering with Scaled Features")
> x <- matrix(rnorm(30 * 3), ncol = 3)
> dd <- as.dist(1 - cor(t(x)))
> plot(hclust(dd, method = "complete"),
main = "Complete Linkage with Correlation-Based Distance",
xlab = "", sub = "")

```

Plots:-
```

| Files | Plots | Pac | ges | Help | Viewer | Presentation |  |  | $\square$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\square$ | $\oplus$ Zoom |  | - Export - |  | $\otimes$ | 2 | - Publish | $\cdot \mathrm{C}$ |  |

```

K-Means Clustering Results with K : K-Means Clustering Results with K :



Findings:-
The code uses the elbow method to determine the optimal number of clusters for the USArrests dataset. This involves plotting the WSS against the number of clusters and selecting the point where the decrease in WSS begins to level off. In this case, the elbow occurs at \(\mathrm{k}=3\), so k -means clustering is performed with centers=3. The resulting cluster centers show the average values of each variable for each cluster.

3 )
Code:-
library(ISLR2)
nci.labs <- NCI60\$labs
nci.data <- NCI60\$data
dim(nci.data)
nci.labs[1:4]
table(nci.labs)
pr.out \(<-\operatorname{prcomp}(\) nci.data, scale \(=\) TRUE \()\)
Cols <- function(vec) \{
cols <- rainbow(length(unique(vec)))
```

return(cols[as.numeric(as.factor(vec))])
}
par(mfrow =c(1, 2))
plot(pr.out$x[, 1:2], col = Cols(nci.labs), pch = 19,
xlab = "Z1", ylab = "Z2")
plot(pr.out$x[, c(1, 3)], col = Cols(nci.labs), pch = 19,
xlab = "Z1", ylab = "Z3")
summary(pr.out)
plot(pr.out)
pve <- 100* pr.out$sdev^2 / sum(pr.out$sdev^2)
par(mfrow =c(1,2))
plot(pve, type = "o", ylab = "PVE",
xlab = "Principal Component", col = "blue")
plot(cumsum(pve), type = "o", ylab = "Cumulative PVE",
xlab = "Principal Component", col = "brown3")
sd.data <- scale(nci.data)
par(mfrow =c(1,3))
data.dist <- dist(sd.data)
plot(hclust(data.dist), xlab = "", sub = "", ylab = "",
labels = nci.labs, main = "Complete Linkage")
plot(hclust(data.dist, method = "average"),
labels = nci.labs, main = "Average Linkage",
xlab = "", sub = "", ylab = "")
plot(hclust(data.dist, method = "single"),
labels = nci.labs, main = "Single Linkage",
xlab = "", sub = "", ylab = "")
hc.out <- hclust(dist(sd.data))
hc.clusters <- cutree(hc.out, 4)
table(hc.clusters, nci.labs)
par(mfrow =c(1,1))

```
plot(hc.out, labels \(=\) nci.labs)
abline( \(\mathrm{h}=139\), col \(=\) "red" \()\)
set.seed(2)
km.out <- kmeans(sd.data, 4, nstart \(=20\) )
km.clusters <- km.out\$cluster
table(km.clusters, hc.clusters)
hc.out <- hclust(dist(pr.out\$x[, 1:5]))
plot(hc.out, labels \(=\) nci.labs,
main \(=\) "Hier. Clust. on First Five Score Vectors")
table(cutree(hc.out, 4), nci.labs)

Output:-



\(>\operatorname{par}(m f r o w=c(1,1))\)
\(>\mathrm{plot}(\mathrm{hc}\). out, labels \(=\) nci.1abs)
> abline(h = 139, col = "red")
> hc.out
Ca11:
hclust(d = dist(sd.data))
Cluster method : complete
Distance : euclidean
Number of objects: 64
> set. seed(2)
\(>\mathrm{km}\). out <- kmeans(sd.data, 4, nstart \(=20\) )
\(>\) km.clusters <- km.out\$cluster
> table(km.clusters, hc.clusters)
hc.clusters
km.clusters \(11 \begin{array}{llll}1 & 2 & 3\end{array}\)
\(\begin{array}{lllll}1 & 11 & 0 & 0 & 9\end{array}\)
\(\begin{array}{lllll}2 & 20 & 7 & 0 & 0\end{array}\)
\(\begin{array}{lllll}3 & 9 & 0 & 0 & 0\end{array}\)
\(\begin{array}{lllll}4 & 0 & 0 & 8 & 0\end{array}\)
\(>\) hc. out <- hclust(dist(pr.out \(\$ \times[, 1: 5])\) )
\(>\mathrm{p}\) lot (hc. out, labels = nci.1abs,
\(+\quad\) main \(=\) "Hier. Clust. on First Five Score Vectors")
> table(cutree(hc.out, 4), nci.labs)
nci. 1 abs
BREAST CNS COLON K562A-repro K562B-repro LEUKEMIA MCF7A-repro MCF7D-repro MELANOMA NSCLC
\begin{tabular}{llllllllll}
1 & 0 & 2 & 7 & 0 & 0 & 2 & 0 & 0 & 1 \\
2 & 5 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 7 \\
3 & 0 & 0 & 0 & 1 & 1 & 4 & 0 & 0 & 0 \\
4 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
nci.labs & & & & & & 0 & 0
\end{tabular} OVARIAN PROSTATE RENAL UNKNOWN
\begin{tabular}{rllll}
1 & 5 & 2 & 7 & 0 \\
2 & 1 & 0 & 2 & 1 \\
3 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & 0
\end{tabular}

Plots:-
```

Files Plots Packages Help Viewer Presentation

```




\begin{tabular}{|l|l|l|l|l|l|}
\hline Files & Plots & Packages & Help & Viewer & Presentation
\end{tabular}

\section*{Hier. Clust. on First Five Score Vectors}

dist(pr.out\$x[, 1:5])
hclust (*, "complete")```

